

## Effect of seismic input on hydrodynamic forces acting on gravity dams

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### ABSTRACT

A 2D constant boundary element reservoir model is used to obtain hydrodynamic forces due to harmonic horizontal and vertical excitations. Energy dissipation due to both infinite radiation and damping along the bottom of the reservoir has been incorporated. The Fast Fourier transformation (FFT) technique is then used to find a response to a given seismic input through the frequency domain analysis. This gives the time history of the resultant hydrodynamic force acting on the dam face. The results indicate that the total response in hydrodynamic forces depends not only on the geometry of the reservoir but also on the nature of the seismic input (frequency content, ground acceleration and duration of strong motion). Critical depths, at which maximum reservoir response occurs, are identified for various seismic excitations.

### INTRODUCTION

A number of studies have been carried out on the seismic response of 2D gravity dam-reservoir-foundation deterministic models using both finite element and boundary element methods (Hall and Chopra, 1980; Hanna and Humar, 1982; Liu, 1984; Humar and Jablonski, 1988). Some attempts have been made to introduce non-deterministic models (Cheng, Yang and Niu, 1990). However, little attention has been given to the effect of different seismic inputs on hydrodynamic forces acting on gravity dams.

The analytical 2D model of the system, based on the model first introduced with the FEM (Hall and Chopra, 1980), is used in this study. The energy loss in the outgoing waves has been modelled by assuming that beyond a certain distance upstream of the dam the reservoir has a uniform rectangular shape. Boundary element discretization is limited to the irregular region of the reservoir in the vicinity of the dam face. For the regular but infinite region a one-dimensional finite element solution is employed. The effect of the reservoir foundation damping is also included based on a simplified boundary condition which models the absorption of the reservoir bottom using a one-dimensional compressive wave propagation model (Hall and Chopra, 1980). This model has been incorporated into a boundary element solution for small vibrations of the reservoir (Humar and Jablonski, 1988). This paper presents the results of more recent study on the application of BEM

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and the Fast Fourier transformation technique to the seismic analysis of a 2D infinite reservoir bounded by a gravity dam. At first the given system is solved for horizontal and vertical harmonic excitations with the use of the previously developed and recently modified computer program BEMC2D (Humar and Jablonski, 1988). Then, the Fast Fourier transformation technique is used to obtain the combined response of the system for horizontal and vertical components of two different earthquake acceleration records: the Saguenay earthquake of 1988 and the El Centro earthquake of 1940. The objective of this study is to show the effect of seismic input on hydrodynamic response of the reservoir. The critical depth for obtaining maximum reservoir response is identified for the above mentioned earthquake records.

### BOUNDARY ELEMENT FORMULATION FOR HARMONIC RESPONSE

The details of the development of the boundary element model of the reservoir can be found in the earlier publications (Liu, 1984; Humar and Jablonski, 1988). Only a few of the important assumptions and boundary conditions are summarized here.

The concrete gravity dam and reservoir are modelled as a two-dimensional system in which only planar vibrations are considered. The base of the dam and the bottom of the reservoir may undergo the prescribed acceleration history due to horizontal and vertical components of the ground motion. Dam-foundation interaction effects are not included in this model, but the bottom of the reservoir may be flexible or almost rigid. Small vibration theory is used to solve the problem. The governing equations are the linearized Navier-Stokes equations, which are valid for non-viscous and compressible water. For harmonic motion of the dam and foundation, uniformly distributed along the dam and the reservoir bottom, the Navier-Stokes equations are reduced to the Helmholtz equation:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + k^2 p = 0 \quad (1)$$

where  $k = \omega/c$  the reservoir wave number,  $p$  = the water pressure,  $\omega$  = the excitation frequency,  $c$  = velocity of sound in water.

Along the reservoir bottom, a special boundary condition originally developed by Hall and Chopra (1980) is used,

$$\frac{\partial p}{\partial n}(s', \omega) = -\frac{w}{g} a_n(s') - i \omega \gamma p(s', \omega) \quad (2)$$

in which  $s'$  = the coordinate along the reservoir bottom,  $\gamma = w/(w_r c_r)$  = the damping coefficient of the reservoir bottom,  $w_r$  = specific weight of the bottom material,  $w$  = specific weight of water,  $c_r$  = compression wave velocity, travelling in the direction normal to the reservoir bottom interface; and  $a_n(s')$  = the harmonic excitation amplitude along the outward normal.

Damping of the reservoir bottom can be expressed with help of a wave reflection coefficient  $\alpha_R$  by

$$\gamma = \frac{1}{c} - \frac{\alpha_R}{1 + \alpha_R} \quad (3)$$

For a rigid reservoir bottom,  $\alpha_R = 1.0$ , and  $\gamma = 0$ . i.e. damping is absent. For  $\alpha_R = 0$ , and  $\gamma = 1/c$ , this represents full damping. The wave reflection coefficient may be expressed in the following form:

$$\alpha_R = \frac{1 - w_c/(w_r c_r)}{1 + w_c/(w_r c_r)} \quad (4)$$

Along the transmitting boundary, the boundary condition has been derived based on the one-dimensional finite element model of the infinite region (Hall and Chopra, 1980; Humar and Jablonski, 1988). It has been successfully used in both the FEM and BEM formulations. For a horizontal and/or vertical harmonic ground motion this leads to a solution of an eigenvalue problem. This solution is then coupled with the boundary element solution for the finite irregular region of the reservoir adjacent to the dam. Compatibility of the pressure and pressure gradients is then imposed at the interface of the regular and irregular regions. The reservoir damping in the infinite region is also included.

#### RESPONSE TO AN ARBITRARY GROUND MOTION

When the excitation history is given in terms of a function of the acceleration  $a(t)$  and  $\tilde{F}(\omega)$  represents the harmonic response of the system to a unit acceleration,  $e^{i\omega t}$ , the so-called harmonic synthesis (superposition of all responses) may be expressed in the following form:

$$F(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(\omega) \tilde{F}(\omega) e^{i\omega t} d\omega \quad (5)$$

where

$$A(\omega) = \int_{-\infty}^{\infty} a(t) e^{-i\omega t} dt \quad (6)$$

Equation 6 is the Fourier Transform of the acceleration function  $a(t)$ , while Eq. 5 is the Inverse Fourier Transform of the product of the frequency functions  $A(\omega)$  and  $\tilde{F}(\omega)$ . The continuous expressions given by Eqs. 5 and 6 can be converted into finite sums in the following form:

$$\{F(m\Delta t)\} = \frac{\Delta\omega}{2\pi} \sum_{n=0}^{N-1} \{A(n\Delta\omega) \tilde{F}(n\Delta\omega)\} e^{i\frac{2\pi nm}{N}} \quad (7)$$

$$\{A(n\Delta\omega)\} = \Delta t \sum_{m=0}^{N-1} \{a(m\Delta t)\} e^{-i\frac{2\pi nm}{N}} \quad (8)$$

Then, the Fast Fourier Transformation (FFT) technique is used to evaluate Eqs. 7 and 8. A computer program has been developed to calculate the response of the linear system subjected to a given excitation (Jablonski, 1990).

### PARAMETRIC STUDY OF RESERVOIR RESPONSE

Two sets of parametric studies were carried out: 1. variation of reservoir boundary conditions; 2. variation of reservoir depth. For both sets of calculations 16 seconds of ground motions were used from the Saguenay earthquake of November 25, 1988, and the 1940 El Centro earthquake in California. Further details of the motions are given in Table 1.

#### Variation of reservoir damping and shape

Various reservoir bottom absorption conditions were chosen, as represented by the reflection coefficients  $\alpha_R = 0.975, 0.75$ , and  $0.50$ . The reservoir has a water depth  $H$  of  $100\text{ m}$  at the dam. One reservoir is of rectangular shape of infinite length. For the other two, the reservoir bottom slopes from a depth of  $H_D = 100\text{ m}$  at the dam to  $H_I = 50\text{ m}$  over a distance  $L$  of  $200\text{ m}$  from the dam; thereafter the water depth is constant at  $50\text{ m}$ . The rectangular reservoir has a fundamental resonance frequency of  $3.6\text{ Hz}$ , the sloping reservoir,  $4.4\text{ Hz}$ . The horizontal and vertical components of seismic ground motions were applied both individually and simultaneously in order to observe the influence of excitation direction. The results of the numerical study are presented in Table 2, giving the ratio of the peak resultant hydrodynamic force to the hydrostatic force (maximum positive and negative values) on the vertical dam face.

#### Variation of reservoir depth

Rectangular reservoirs of depths  $H = 36\text{ m}, 77.6\text{ m}$ , and  $113.8\text{ m}$  were selected so that the resonance frequency of the reservoir coincides with specific peaks or valleys in the vertical response spectra of the two seismic motions shown in Figs. 1 and 2. No horizontal motion was applied for this set of calculations. The resonance frequencies are  $10\text{ Hz}, 4.6\text{ Hz}$ , and  $3.2\text{ Hz}$  corresponding to above water depths. The results of resultant peak hydrodynamic force ratios on the dam for two reservoir bottom reflection coefficients,  $\alpha_R = 0.975$  and  $0.75$ , are shown in Table 3. Representative time variations of the resultant hydrodynamic force ratios are shown in Figs. 3 and 4.

### DISCUSSION OF RESULTS

From Table 2, the peak force ratios for the rectangular reservoir are seen to be primarily influenced by excitation in the vertical direction. For the sloping reservoir bottom, however, excitation in the horizontal direction becomes more significant, generally exceeding the results from the vertical component for the El Centro motion. This effect depends also on the spectral content of the vertical and horizontal components of ground motion as can be seen by comparing the results for the same geometry, Nos. 2 and 3 in Table 2. The

Saguenay results, No. 2, are always higher for the vertical excitation than the horizontal or the combined ones, whereas for the El Centro excitation, No. 3, the horizontal one predominates. In all cases, the maximum peak from the combined horizontal and vertical excitation is close to the maximum of either of the two individual excitations.

The reflection coefficient for the reservoir bottom is seen to have a significant effect on the peak pressure on the dam. For all results in Table 2, reductions in peak force ratios by factors of 3 to 4 occur for  $\alpha_R = 0.75$  as compared to  $\alpha_R = 0.975$ . Only small further reductions occur for  $\alpha_R = 0.5$ . The absorption properties of the reservoir bottom are therefore of major importance in relation to the peak dynamic pressures on the dam.

The peak resultant pressures as a function of the variation of the reservoir depth are presented in Table 3. For the shallowest reservoir, Case 1 and for  $\alpha_R = 0.975$ , the peak hydrodynamic force from the El Centro motion exceeds substantially the hydrostatic force. This case coincides with a peak in the response spectrum curve shown in Fig. 2. For the lower reflection coefficient  $\alpha_R = 0.75$ , the peaks of hydrodynamic forces then become less than the hydrostatic ones, but are still of substantial amplitude. The results for the Saguenay earthquake are quite moderate in comparison. For Case 2 in Table 3, the results for Saguenay earthquake exceed those from the El Centro earthquake for the higher  $\alpha_R$ . For Case 3, the deepest reservoir, the El Centro excitation produces larger pressures than Saguenay. As was the case for the results presented in Table 2, substantial reductions in peak hydrodynamic forces occur, by factors of 2 to 3, when the reflection coefficient changes from  $\alpha_R = 0.975$  to 0.75.

#### CONCLUSIONS

1. The damping characteristics of the reservoir bottom have a significant effect on the peak hydrodynamic forces on the dam caused by seismic excitations.
2. The geometry of the reservoir affects the relative contributions to the total hydrodynamic pressure arising from horizontal and vertical excitations.
3. For obtaining maximum pressures on the dam, the critical depth of the reservoir varies for different seismic records and may be derived with the help of response spectra. For the parameters chosen in this study, the seismically induced hydrodynamic pressures acting on gravity dams can be a considerable fraction of static water pressure, and in certain cases, can exceed it.

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Table 1. Characteristics of earthquake records

Earthquake Record	Component	Peak Acceleration	Notes
M5.7, Saguenay, November 11, 1988 Site 16, Chicoutimi, Quebec	Vertical	$A_V = -1.005$	High frequency content
	Horizontal, $214^\circ$	$A_H = 1.045$	
M6.6, Imperial Valley, May 18, 1940 El Centro, California	Vertical	$A_V = 2.063$	Strong shaking
	S-E Horizontal	$A_H = 3.417$	

Table 2. Maximum amplitudes of hydrodynamic forces  
for various reservoirs

No.	Reservoir Model	Reservoir Fundamental Frequency [Hz]	Earthquake Motion	$\alpha_R = 0.50$			$\alpha_R = 0.75$			$\alpha_R = 0.975$		
				Horiz.	Vert.	Comb.	Horiz.	Vert.	Comb.	Horiz.	Vert.	Comb.
1	Rectangular Infinite $H = 100 \text{ m}$	3.6	Saguenay, 1988	0.031	0.080	0.079	0.038	0.115	0.119	0.049	0.164	0.180
				-0.039	-0.063	-0.065	-0.039	-0.109	-0.114	-0.044	-0.179	-0.196
2	Inclined Infinite $H_D = 100 \text{ m}$ $H_I = 50 \text{ m}$	4.4	Saguenay, 1988	0.016	0.049	0.049	0.021	0.095	0.091	0.045	0.245	0.231
				-0.019	-0.042	-0.041	-0.023	-0.091	-0.092	-0.042	-0.253	-0.233
3	Inclined Infinite $H_D = 100 \text{ m}$ $H_I = 50 \text{ m}$	4.4	El Centro, 1940	0.144	-0.056	0.132	0.174	0.121	0.165	0.343	0.304	0.363
				-0.164	-0.089	-0.143	-0.172	-0.121	-0.180	-0.300	-0.300	-0.396

Table 3. Maximum amplitudes of hydrodynamic forces on dam from vertical excitation in rectangular reservoirs

Case	Reservoir Depth H [m]	Reservoir Fundamental Frequency [Hz]	Earthquake Motion	Ratio Hydrodynamic to Static Force	
				$\alpha_R = 0.975$	$\alpha_R = 0.75$
1	36	10	Saguenay	0.269	0.128
				-0.261	-0.124
			El Centro	1.814	0.689
				-1.610	-0.588
2	77.56	4.6	Saguenay	0.460	0.155
				-0.463	-0.148
			El Centro	0.403	0.179
				-0.420	-0.224
3	113.83	3.2	Saguenay	0.120	0.076
				-0.114	-0.066
			El Centro	0.348	0.203
				-0.333	-0.165

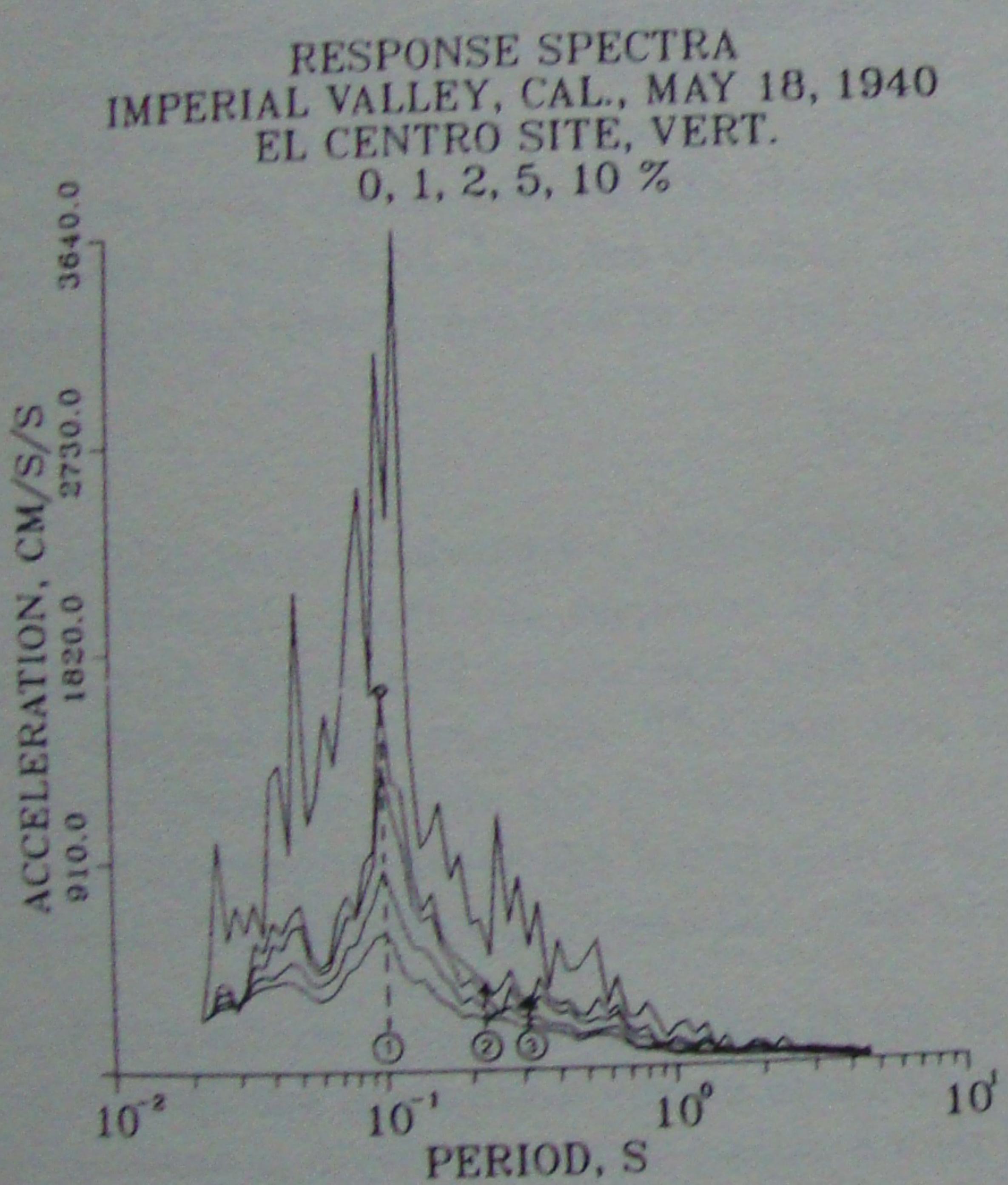


Figure 1. The El Centro Earthquake response spectra, vertical component, showing Cases 1, 2 and 3 of Table 3

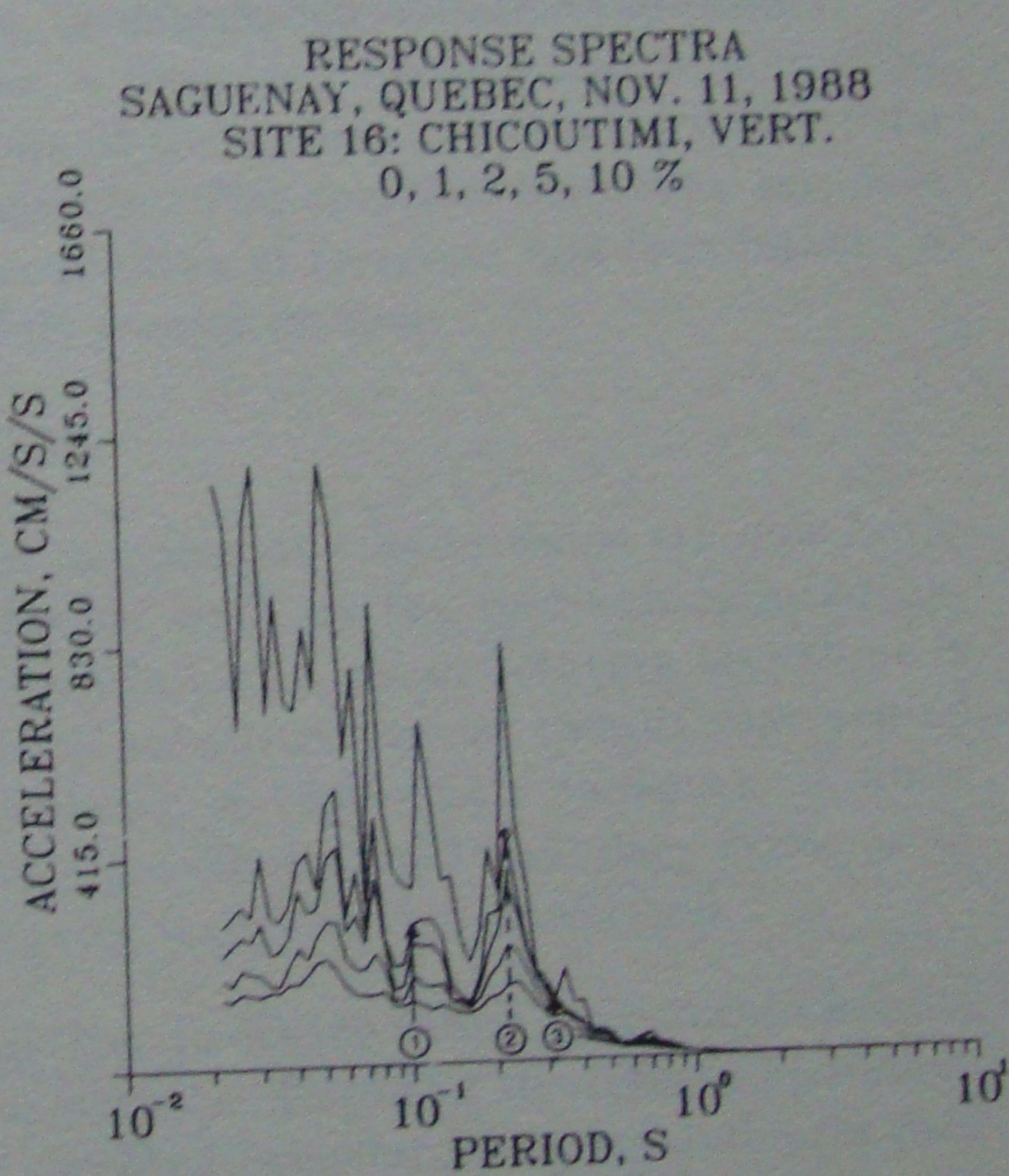


Figure 2. The Saguenay Earthquake response spectra, vertical component, showing Cases 1, 2 and 3 from Table 3

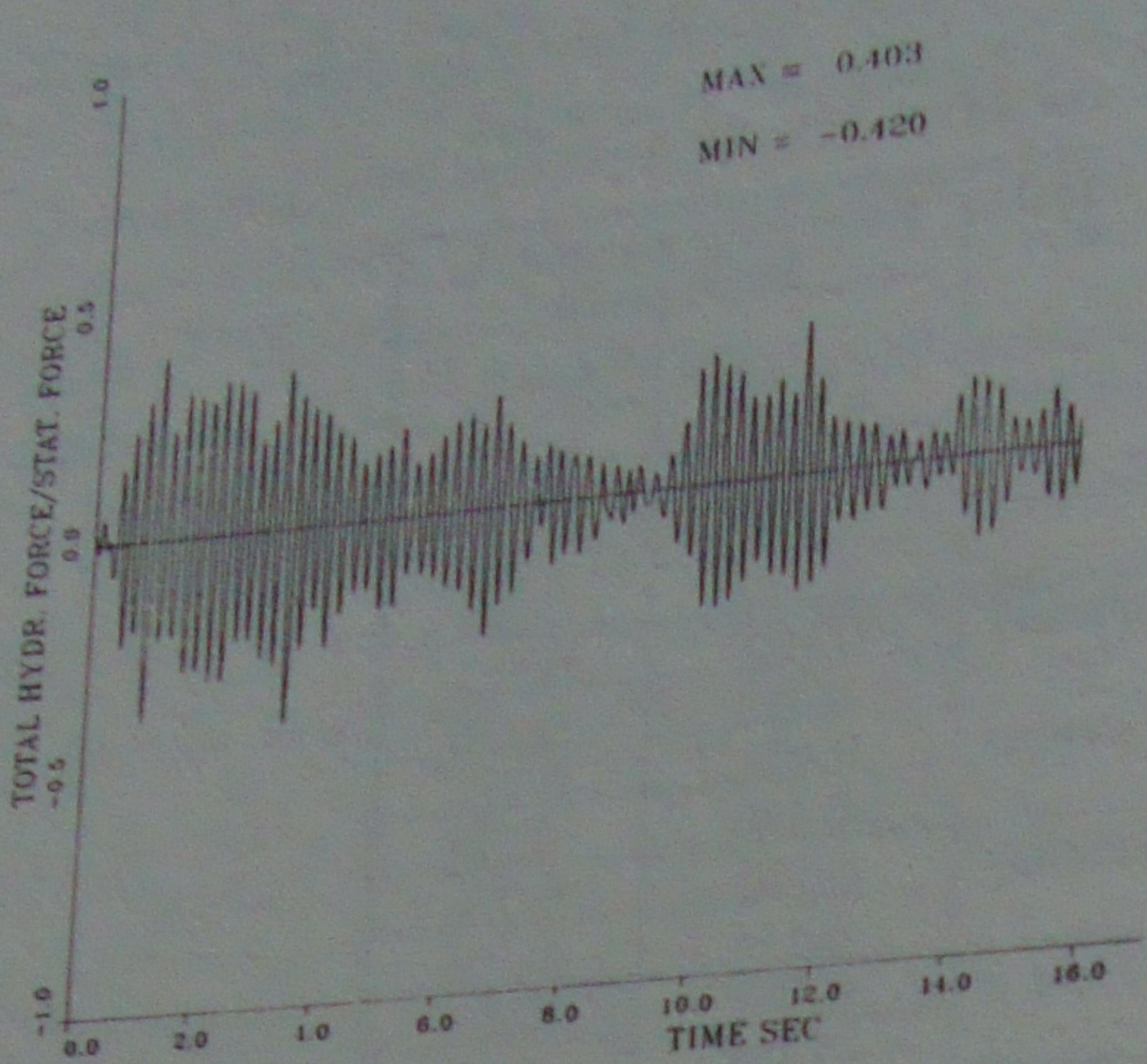


Figure 3. Time history of hydrodynamic force for Case 2,  $H = 77.56\text{ m}$  due to the El Centro Earthquake, vertical component

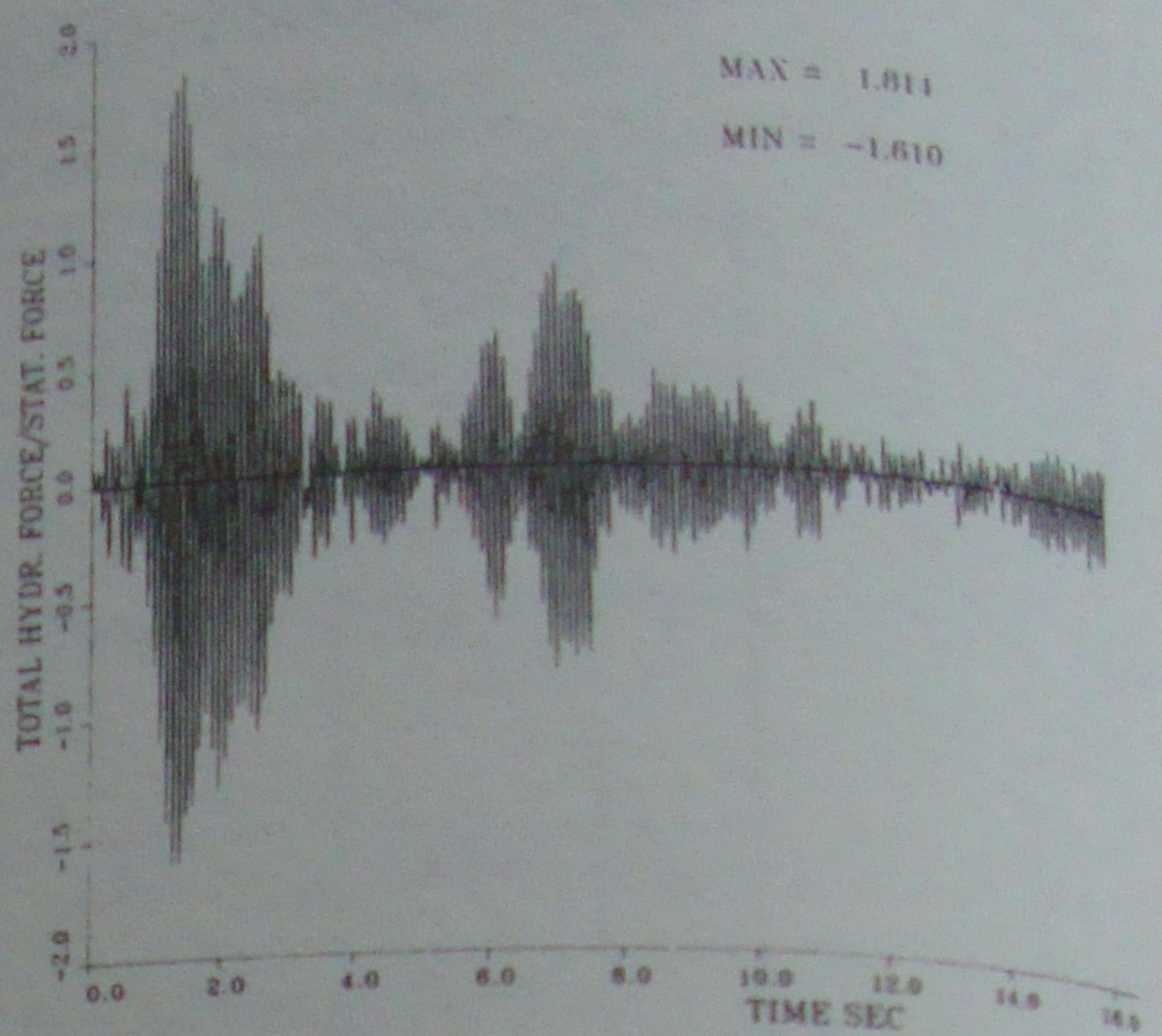


Figure 4. Time history of hydrodynamic force for Case 1,  $H = 36\text{ m}$  due to the El Centro Earthquake, vertical component